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Since the first point can fall in any of the six portions into which the medians divide the triangle, the required probability is $P=6p$.

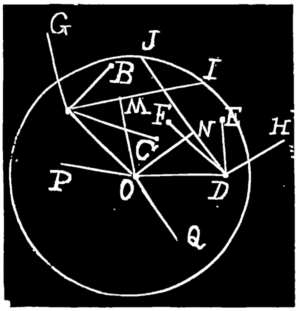
$$\therefore P=\tfrac{6}{27}(\tfrac{1}{3}+\tfrac{5}{9}\log 4)=\tfrac{1}{27}(2+\tfrac{10}{3}\log 4=\tfrac{2}{27}(1+\tfrac{10}{3}\log 2).$$

91. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Six points A, B, C, D, E, F, are taken at random on the surface of a sphere. Find the chance that the plane through A, B, C intersects the plane through D, E, F within the sphere.

Solution by the PROPOSER.

Let AI, DJ be the diameters of the sections of the sphere made by the planes through A, B, C and D, E, F; M and N their centers, O the center of the sphere, OP a line such that AB is parallel to the plane MOP, OQ a line such that DE is parallel to the plane NOQ. Draw AG, DH perpendicular to AI, DJ, respectively.



Let $AO=r$, $\angle AOM=\theta$, $\angle DON=\varphi$, $\angle GAC=\psi$, $\angle GAB=\delta$, $\angle HDF=\mu$, $\angle HDE=\nu$, $\angle MOP=\lambda$, the angle the plane MOP makes with some fixed plane through $OP=\rho$, $\angle NOQ=\gamma$, and the diedral angle $MOQN=\eta$.

An element of surface at A is $4\pi r^2\sin\theta d\theta$; at D, $4\pi r^2\sin\varphi d\varphi$; at C, $4r^2\sin\theta\sin\psi d\psi d\lambda$; at B, $4r^2\sin\theta\sin(\psi-\delta)\sin\lambda\sin\delta d\delta d\rho$; at F, $4r^2\sin\varphi\sin\mu d\mu d\gamma$; at E, $4r^2\sin\varphi\sin(\mu-\nu)\sin\gamma\sin\nu d\nu d\eta$.

The limits of θ are 0 and $\frac{1}{2}\pi$; of φ, 0 and $\frac{1}{2}\pi$; of ψ, 0 and π; of δ, 0 and ψ; of μ, 0 and π; of ν, 0 and μ; of λ, 0 and π; of γ. $\pm(\theta-\varphi)$ and $\theta+\varphi$ (the double sign being taken $+$ when $\theta>\varphi$, and $-$ when $\theta<\varphi$); of ρ, 0 and 2π; of η, 0 and 2π.

Since the whole number of ways six points can be taken in the surface of the sphere is $(4\pi r^2)^6$ the required chance is

$$p=\frac{1}{(4\pi r^2)^6}\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\int_0^{\pi}\int_0^{\pi}\int_0^{\psi}\int_0^{\mu}\int_0^{\pi}\int_{\pm(\theta-\psi)}^{\theta+\psi}\int_0^{2\pi}\int_0^{2\pi} 4\pi r^2\sin\theta.d\theta 4\pi r^2$$

$$\sin\varphi d\varphi.4r^2\sin\theta\sin\psi d\psi d\lambda.4r^2\sin\varphi\sin\mu d\mu d\gamma.4r^2\sin\theta\sin(\psi-\delta)\sin\lambda\sin\delta d\delta d\rho$$

$$\times 4r^2\sin\varphi\sin(\mu-\nu)\sin\gamma\sin\nu d\nu d\gamma$$

$$=\frac{2}{\pi^3}\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\int_0^{\pi}\int_0^{\pi}\int_0^{\psi}\int_0^{\mu}\int_0^{\pi}\int_{\pm(\theta-\psi)}^{\theta+\psi}\int_0^{2\pi}\sin^3\theta\sin^3\varphi\sin\psi\sin\mu\sin(\psi-\delta)$$

$$\sin(\mu-\nu)\sin\delta\sin\nu\sin\lambda\sin\gamma d\theta d\varphi d\psi d\mu d\delta d\nu d\gamma d\rho$$

$$=\frac{4}{\pi^2}\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\int_0^{\pi}\int_0^{\pi}\int_0^{\psi}\int_0^{\mu}\int_0^{\pi}\int_{\pm(\theta-\psi)}^{\theta+\psi}\sin^3\theta\sin^3\varphi\sin\psi\sin\mu\sin(\psi-\delta)$$

$$\sin(\mu-\nu)\sin\delta\sin\nu\sin\lambda\sin\gamma\, d\theta d\varphi d\psi d\mu d\delta d\nu d\lambda d\gamma$$

$$=\frac{8}{\pi^2}\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\int_0^{\pi}\int_0^{\pi}\int_0^{\psi}\int_0^{\mu}\int_0^{\pi}\sin^4\theta\sin^4\varphi\sin\psi\sin\mu\sin(\psi-\delta)\sin(\mu-\nu)$$

$$\sin\delta\sin\nu\sin\lambda\, d\theta d\varphi d\psi d\mu d\delta d\nu d\lambda$$

$$=\frac{16}{\pi^2}\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\int_0^{\pi}\int_0^{\pi}\int_0^{\psi}\int_0^{\mu}\sin^4\theta\sin^4\varphi\sin\psi\sin\mu\sin(\psi-\delta)\sin(\mu-\nu)\sin\delta\sin\nu$$

$$d\theta d\varphi d\psi d\mu d\delta d\nu$$

$$=\frac{8}{\pi^2}\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\int_0^{\pi}\int_0^{\pi}\int_0^{\psi}\sin^4\theta\sin^4\varphi\sin\psi\sin\mu(\sin\mu-\mu\cos\mu)\sin(\psi-\delta)\sin\delta$$

$$d\delta d\varphi d\psi d\mu d\delta$$

$$=\frac{4}{\pi^2}\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\int_0^{\pi}\int_0^{\pi}\sin^4\theta\sin^4\varphi\sin\psi(\sin\psi-\psi\cos\psi)\sin\mu(\sin\mu-\mu\cos\mu)$$

$$d\theta d\varphi d\psi d\mu$$

$$=\frac{3}{\pi}\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\int_0^{\pi}\sin^4\theta\sin^4\varphi\sin\psi(\sin\psi-\psi\cos\psi)d\theta d\varphi d\psi$$

$$=\tfrac{9}{4}\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\sin^4\theta\sin^4\varphi\, d\theta d\varphi=\frac{27\pi}{64}\int_0^{\frac{1}{2}\pi}\sin^4\theta\, d\theta=\frac{81\pi^2}{1024}=(\tfrac{3}{4})^4(\tfrac{1}{2}\pi)^2.$$

NOTE. We may publish a second solution of Problem 90 in the next issue of the MONTHLY. ED. F.

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PROBLEMS FOR SOLUTION.

ARITHMETIC.

140. Proposed by **ALOIS F. KOVARIK**, Instructor in Mathematics and Science, Decorah Institute, Decorah, Iowa.

$\frac{1}{7}=0.\dot{1}4285\dot{7}$; $\frac{1}{14}=0.\dot{0}71428\dot{5}$; $\frac{1}{21}=0.0\dot{0}4761\dot{9}$. Notice that the sum of the figures in each period is equal to 27. This is not true with $\frac{1}{72}$, $\frac{1}{73}$. Is there any general law of which these are special cases, and if so, what is it ?

141. Proposed by **F. P. MATZ. M. Sc., Ph. D.**, Professor of Mathematics and Astronomy in Irving College, Mechanicsburg. Pa.

If the alloy in a half-dollar be 1-13th of the mass, and the coin be worth a cent if it be all alloy, what should be the exact value of the coin if it be all pure silver?

⁂ Solutions of these problems should be sent to B. F. Finkel not later than April 10.